

Identification of Ground Motion Parameters that Control Structural Damage using a Slepian Process Model

Final Report

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Abstract

The ability to predict the behavior of a random process immediately following a level crossing can provide critical engineering design information. In this study the effects of non-Gaussian, non-stationary input on the response of linear and nonlinear structures subject to earthquake excitation is investigated using a Slepian process model. The strong ground motion portion of a suite of earthquake records, which is essentially Gaussian, was extracted from the total ground motion records in order to investigate the effect of non-Gaussian input on the response predictions. In this study the attention was directed at the prediction of extremes for linear, and non-linear hysteretic oscillators. Predictions for the linear oscillators were found to be good, while predictions for the highly nonlinear oscillators were poorer until an approximate correction for the change in equilibrium position experienced during the motion response behavior was applied. Based upon the experience gained from these examples, a three-member frame, whose beam-to-column connections were modeled as tri-linear hysteretic was investigated. The structural reliability, which is a basic measure of the safety reserve in a system, was bounded using the Slepian prediction of the frames response.

Non-Technical Summary

Ground motions that appear to be of approximately the same intensity level often cause very different levels of damage in engineered structures, even when their designs are very similar. This is because there are certain characteristics within each ground motion that cause more damage to certain types of structures. This study used a statistical approach that was originally developed for use in audio signal processing to examine the damaging effect of a suite of ground motion records using structural models of varying complexity. The most complex model was a reinforced concrete frame and the safety of the frame during earthquake excitation was readily determined by applying the Slepian procedure.

1. Introduction

In order to understand ground motion and begin to link it with structural damage, a comprehensive description of the ground motion is needed. Ground motion is a random process, which can, if assumed stationary, be generated as the sum of infinitesimal sine waves with each having its own amplitude and phase. However, ground motion is a complex nonstationary phenomena that generally requires addressing in the time domain. The extremes of a random process play an important role in most engineering design problems including those involving the estimation of structural damage accumulation. One very important problem in earthquake engineering, which to a large extent remains unsolved, has been the prediction of the behavior of a random process after it crosses some predefined level. The importance of understanding this problem can not be understated since it would provide a direct link to modeling the extremes and eventually damage prediction. One predictive model, developed in the early 1960's (Kac and Slepian 1959, Slepian 1961, 1962), has had success in predicting the extreme behavior of random Gaussian processes, but has had only moderate application to earthquake engineering problems and virtually no application to nonstationary random processes.

1.1 The Slepian Model

A Slepian process model, introduced by Bell Laboratory's physicist David Slepian (1961, 1962), is used to describe the conditional behavior of a stochastic process following events defined by level or threshold crossings. There are two types of Slepian models. The Type I model is used to describe the behavior of a process immediately following a level crossing, and the Type II model is used to describe a process immediately following an extreme event. The work presented herein focuses on the Type I model. This type of model describes the extreme behavior of a process in terms of the covariance of the underlying process and the statistical distribution of the first derivative at level crossings. It is conditioned on the threshold level with the assumption that the derivatives at the point of upward level crossing follows the Rayleigh distribution. One very powerful feature of this modeling process is that one can easily vary the crossing level to obtain new predictions without any additional computation.

1.2 Quantifying Damage

Earthquakes are a powerful force of nature and will continue to occur in densely populated urban areas all over the world. As seismologists understand more and more about earthquake mechanisms, earthquake engineers are beginning to understand how to optimize the conflicting design objectives of minimizing deflections while maximizing energy dissipation. The present study seeks to work toward finding a new way to identify which ground motions are more damaging than others. Engineers and scientists tend to quantify almost everything in order to give a process some type of order. This is often done in terms of a parameter or index that relate the energy or some other quantity of the ground motion to the observed structural damage occurrence. In addition, it is often

advantageous for engineers to characterize the ground motion statistically and attempt to relate these quantities to structural damage.

Overall structural damage to a building is of primary importance since it is directly related to economic losses and loss of life. Scale criteria have been used to classify damage grades based on on-site inspection information (Cabanias et al. 1997). Seismic damage indexes can quantitatively describe damage to a structure or its components, and provide an important link between analysis and acceptance criteria. Damage indexes for seismically detailed structural elements are generally based on element forces, irrecoverable deformations, dissipated energy or some combination of these quantities. It is said that the majority of structural damage indexes are related to the irreversible damage done to structures in the post-yield range. An energy dissipation index was developed previously by Darwin and Nmai (1986) for beams under cyclic loading. There also exist damage models that are based on stiffness degradation under reversed cyclic loading (Banon et al. 1981; Roufaiel and Meyer 1987). Mehanny and Deierlein (2001) proposed a new cumulative ductility damage index for structural elements that are commonly used in steel/concrete composite construction. Park and Ang (1985) expressed seismic structural damage as a linear combination of the damage caused by excessive deformation and that contributed by the repeated cyclic loading effect that occurs during ground shaking. They represented this in terms of a damage index, D , which can be calibrated to dynamic and monotonic test results. As a result of a robust regression analysis using years of experimental data from various research studies, their model is mechanistic for reinforced concrete structures. Mathematically, the Park-Ang damage index can be expressed as

$$D = \frac{\delta_M}{\delta_u} + \frac{\psi}{Q_y \delta_u} \int dE \quad (1)$$

in which δ_M = maximum deformation under earthquake; δ_u = ultimate deformation under monotonic loading; Q_y = calculated yielding strength; dE = incremental absorbed hysteric energy; ψ = model calibration parameter. This damage model was used as a measure of structural damage in the present study due to its acceptance as a credible measure in reinforced concrete structural engineering applications.

2. Mathematical Formulation

2.1 Derivation of the Type I Slepian Process Model

The Slepian model can be derived several ways but here we follow and expand upon the derivation by Randrup-Thomsen and Ditlevsen (1997). In the derivation we seek to linearly regress the random process $X(t)$ on the vector $[X_1(0), \dot{X}_1(0), \ddot{X}_1(0)]$ that can be more generally interpreted as a vector consisting of the specified crossing level, the derivative of the process at that crossing level, and the second derivative of that process at a critical point. This derivation then involves matrices that can be treated analytically using basic matrix methods; however a few preliminaries will prove helpful in understanding this approach.

Consider the general matrix A whose inverse, A^{-1} , one would like to obtain. To accomplish this several intermediate matrix manipulations are required. Here we focus the discussion on the solution for the case when A is a 3x3 matrix. It follows then that

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (2)$$

$$A^{-1} \equiv \frac{\text{adj } A}{\det A} = \frac{\text{adj } A}{|A|} \quad (3)$$

Thus the determinant and the adjoint of the matrix must be obtained. The determinant of the matrix A can be expressed as

$$\det A \equiv |A| = \sum_{r=1}^n a_{rs} |A_{rs}| \quad (4)$$

$$\begin{aligned} |A| &= (-1)^{1+1} a_{11} |A_{11}| + (-1)^{1+2} a_{12} |A_{12}| + (-1)^{1+3} a_{13} |A_{13}| \\ &= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ &= a_{11} (a_{22}a_{33} - a_{23}a_{32}) - a_{12} (a_{21}a_{33} - a_{23}a_{31}) + a_{13} (a_{21}a_{32} - a_{22}a_{31}) \end{aligned} \quad (5)$$

The adjoint of the matrix, $\text{adj } A \equiv [(-1)^{i+j} |\text{eliminating the } i\text{th row and } j\text{th column}|]$ and it follows then that for the matrix A one obtains

$$\text{adj } A = \begin{bmatrix} (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & (-1)^{1+2} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & (-1)^{1+3} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ (-1)^{2+1} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & (-1)^{2+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (-1)^{2+3} \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ (-1)^{3+1} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & (-1)^{3+2} \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & (-1)^{3+3} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{bmatrix} \quad (6)$$

The linear regression was expressed in terms of the conditional covariance by Randrup-Thomsen and Ditlevsen (1997) as

$$E[X_i|Y] = \text{Cov}[X_i|Y] \text{Cov}[Y'|Y]^{-1} Y' \quad (7)$$

where, for compactness in the derivation to follow the following definitions have been used

$$X_i = X_i(\tau) \quad (8)$$

$$Y = [X_1, \dot{X}_1, \ddot{X}_1] = [X_1(0), \dot{X}_1(0), \ddot{X}_1(0)] \quad (9)$$

In order to evaluate equation (8) the process is divided into as sequence of evaluation steps. Beginning with the first term on the right-hand side of equation (8) one obtains the following

$$\begin{aligned} Cov(X_i, Y) &= Cov(X_i(\tau), [X_1, \dot{X}_1, \ddot{X}_1]) \\ &= \begin{bmatrix} R_{X_i X_1} & R_{\dot{X}_i X_1} & R_{\ddot{X}_i X_1} \end{bmatrix} \\ &= \begin{bmatrix} R_{1i} & -\dot{R}_{1i} & \ddot{R}_{1i} \end{bmatrix} \end{aligned} \quad (10)$$

Next we obtain the expression for the covariance of the second term on the right-hand side of equation (8) in terms of the spectral moments of the random process. This evaluation yields the following matrix

$$Cov[Y' | Y]_{\tau=0} = \begin{bmatrix} R_{X_1 X_1}(0) & -R_{X_1 \dot{X}_1}(0) & R_{X_1 \ddot{X}_1}(0) \\ -R_{\dot{X}_1 X_1}(0) & -R_{\dot{X}_1 \dot{X}_1}(0) & R_{\dot{X}_1 \ddot{X}_1}(0) \\ R_{\ddot{X}_1 X_1}(0) & R_{\ddot{X}_1 \dot{X}_1}(0) & R_{\ddot{X}_1 \ddot{X}_1}(0) \end{bmatrix} = \begin{bmatrix} \lambda_0 & 0 & -\lambda_2 \\ 0 & \lambda_2 & 0 \\ -\lambda_2 & 0 & \lambda_4 \end{bmatrix} \quad (11)$$

From equation (3) it follows that the evaluation of the inverse of this covariance function is obtained in two steps, beginning with the evaluation of the determinant

$$|Cov[Y' | Y]_{\tau=0}| = \begin{vmatrix} \lambda_0 & 0 & -\lambda_2 \\ 0 & \lambda_2 & 0 \\ -\lambda_2 & 0 & \lambda_4 \end{vmatrix} \quad (12)$$

It follows then that

$$|Cov[Y' | Y]_{\tau=0}| = \lambda_0(\lambda_2 \lambda_4 - 0) - 0 + (-\lambda_2)(0 - (\lambda_2)(-\lambda_2)) = \lambda_0 \lambda_2 \lambda_4 - \lambda_2^3 \quad (13)$$

and,

$$\frac{1}{|Cov[Y' | Y]_{\tau=0}|} = \frac{1}{\lambda_2 \lambda_4 (\lambda_0 - \lambda_2^2 / \lambda_4)} \quad (14)$$

The evaluation of the adjoint is straight forward and yields

$$adj Cov[Y'|Y]_{\tau=0} = \begin{bmatrix} \lambda_2 \lambda_4 & 0 & \lambda_2^2 \\ 0 & \lambda_0 \lambda_4 - \lambda_2^2 & 0 \\ \lambda_2^2 & 0 & \lambda_0 \lambda_2 \end{bmatrix} \quad (15)$$

The resulting expression for the inverse of the covariance function when assembled is

$$Cov[Y'|Y]^{-1} = \frac{1}{\lambda_0 - \lambda_2^2 / \lambda_4} \begin{bmatrix} 1 & 0 & \frac{\lambda_2}{\lambda_4} \\ 0 & \frac{\lambda_0}{\lambda_2} - \frac{\lambda_2}{\lambda_4} & 0 \\ \frac{\lambda_2}{\lambda_4} & 0 & \frac{\lambda_0}{\lambda_4} \end{bmatrix} \quad (16)$$

At this point it is now possible to evaluate equation (7), specifically

$$\begin{aligned} E[X_i|Y] &= Cov[X_i|Y] Cov[Y'|Y]^{-1} Y' \\ &= \frac{1}{\lambda_0 - \lambda_2^2 / \lambda_4} \begin{bmatrix} R_{li} & -\dot{R}_{li} & \ddot{R}_{li} \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{\lambda_2}{\lambda_4} \\ 0 & \frac{\lambda_0}{\lambda_2} - \frac{\lambda_2}{\lambda_4} & 0 \\ \frac{\lambda_2}{\lambda_4} & 0 & \frac{\lambda_0}{\lambda_4} \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ \ddot{X}_1 \end{bmatrix} \end{aligned} \quad (17)$$

For the situation where $\lambda_4 \rightarrow \infty$, equation (17) can be simplified

$$E[X_i|Y] = \frac{1}{\lambda_0 - \lambda_2^2 / \lambda_4} \begin{bmatrix} R_{li} & -\dot{R}_{li} & \ddot{R}_{li} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ \dot{X}_1 \\ \ddot{X}_1 \end{bmatrix} \quad (18)$$

Carrying out the matrix multiplication, one obtains an initial form for the expected value of the Type I Slepian model

$$E[X_i(\tau)|X_1, \dot{X}_1, \ddot{X}_1] = \frac{X_1}{\lambda_0} R_{li}(\tau) - \frac{\dot{X}_1}{\lambda_2} \dot{R}_{li}(\tau) \quad (19)$$

At this point the notation can be modified to that used in earlier studies, where the expected values of the level crossing and the slope at level crossing can be expressed as

$$E[X(\tau)|u, z] = \frac{u}{\lambda_0} R_{xx}(\tau) - \frac{E[z]}{\lambda_2} \dot{R}_{xx}(\tau) \quad (20)$$

For a Gaussian process the derivative process of the slope at crossing is known to be a Rayleigh distributed process of the form (Evans, Hastings and Peacock 2000)

$$p(z) = \frac{z}{b^2} e^{-\frac{1}{2}\left(\frac{z}{b}\right)^2} \quad (21)$$

where, the mode b of the derivative process and the expected value of the process is

$$E[z] = b \sqrt{\frac{\pi}{2}} \quad (22)$$

and, the mode can be approximated using the expression

$$b \sim \frac{1}{2N} \sum_{i=1}^N z_i^2 \quad (23)$$

Finally, one obtains the Type I Slepian model used in this research study

$$E[X(\tau)|u, z] = \frac{u}{\lambda_0} R_{xx}(\tau) - \sqrt{\frac{\pi}{2}} \frac{b}{\lambda_2} \dot{R}_{xx}(\tau) \quad (24)$$

The model provides an estimate of the expected behavior of the process above a specified crossing level. It can be observed that once the covariance functions, the mode and the spectral moments of the process have been established, the model can also be used to predict behavior above other crossing level with a minimum of additional computation.

Using this same basic formulation one can obtain the Type II Slepian model that deals with the estimation of the process after a maximum. Beginning with an appropriate form of equation (17) one can write

$$E[X_i|Y] = \frac{1}{\lambda_0 - \lambda_2^2 / \lambda_4} \begin{bmatrix} R_{li} & 0 & \ddot{R}_{li} \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{\lambda_2}{\lambda_4} \\ 0 & \frac{\lambda_0}{\lambda_2} - \frac{\lambda_2}{\lambda_4} & 0 \\ \frac{\lambda_2}{\lambda_4} & 0 & \frac{\lambda_0}{\lambda_4} \end{bmatrix} \begin{bmatrix} X_1 \\ 0 \\ \ddot{X}_1 \end{bmatrix} \quad (25)$$

Performing the matrix multiplications yields

$$E[X(\tau) | X_1, \ddot{X}_1] = \frac{1}{\lambda_0 - \lambda_2^2 / \lambda_4} \left[\left(R_{xx}(\tau) + \frac{\lambda_2}{\lambda_4} \ddot{R}_{xx}(\tau) \right) X_1 + \left(\frac{\lambda_2}{\lambda_4} R_{xx}(\tau) + \frac{\lambda_0}{\lambda_4} \ddot{R}_{xx}(\tau) \right) \ddot{X}_1 \right] \quad (26)$$

This equation can be simplified by introducing other equations relating the spectral moments to other spectral moment equations, these include spectral bandwidth, the mean min-Max rate, and the mean zero crossing period. For example consider using the equation for spectral bandwidth that Cartwright and Longuet-Higgins (1956) defined as

$$\varepsilon^2 = 1 - \frac{\lambda_2^2}{\lambda_0 \lambda_4} \quad (27)$$

This parameter approaches unity as the spectrum approaches white noise. Although the evaluation of λ_0 and λ_2 , the evaluation fourth spectral moment can be problematic. Introducing equation (27) into equation (26) and making some additional substitutions one obtains one form of the Type II Slepian model

$$E[X(\tau) | u, a] = \frac{u}{\lambda_0} \left(\frac{1}{\varepsilon^2} R_{xx}(\tau) + \frac{1 - \varepsilon^2}{\varepsilon^2} \frac{\lambda_0}{\lambda_2} \ddot{R}_{xx}(\tau) \right) + \frac{E[a]}{\lambda_2} \left(\frac{1 - \varepsilon^2}{\varepsilon^2} \left(R_{xx}(\tau) + \frac{\lambda_0}{\lambda_2} \ddot{R}_{xx}(\tau) \right) \right) \quad (28)$$

The expected value of the accelerations at a positive maximum, $E[a]$, can be assumed to be Rayleigh distributed and given by a form and evaluation procedure similar to that of equation (21). The Type II model was presented for completeness and other useful forms of equation (28) can be developed. However, since this model is not used in this study it is sufficient to stop at this point.

2.2 Strong Ground Motion Duration

The ground acceleration associated with an earthquake is a non-stationary, non-Gaussian process, as will be shown in the numerical examples; this is not the case for the strong motion portion of the record as shown in Figure 1. Figure 1 presents a normal PDF superimposed over a normalized histogram for (1) the entire record, and (2) the strong motion portion of the record. Husid et al. (1969) defined the strong motion duration as the time interval required to accumulate a prescribed fraction of the total energy, e.g., 95 percent. Later, Page et al. (1975) defined strong motion duration as the time interval between the first and last peaks equal to or greater than a given level, usually, 0.05g, on an accelerogram. Since there appears to be no universally accepted definition, the strong motion duration definition proposed by Vanmarcke and Lai (1980) will be used. They defined the strong ground motion portion of a record, S_0 , as

$$S_0 = \begin{cases} \left[2 \ln(2S_0/T_0) \right] (I_0/a_{\max}^2) & S_0 \geq 1.36T_0 \\ 2I_0/a_{\max}^2 & S_0 \leq 1.36T_0 \end{cases} \quad (29)$$

where, T_0 is the predominant period of the earthquake motion, a_{max} is the absolute maximum acceleration in the record, and I_0 is the Arias intensity, which can be expressed as

$$I_0 = \int_0^{t_0} a^2(t) dt \quad (30)$$

where, $a(t)$ is the acceleration of the ground in the same units as a_{max} . The upper limit of the integral, t_0 in equation (30) represents the duration of the entire acceleration record. In order to determine the location of the strong ground motion portion, a window having a width equal to S_0 was sequentially moved through the entire record. The strong ground motion portion was selected at the location where the windowed selection of the time series had a maximum variance.

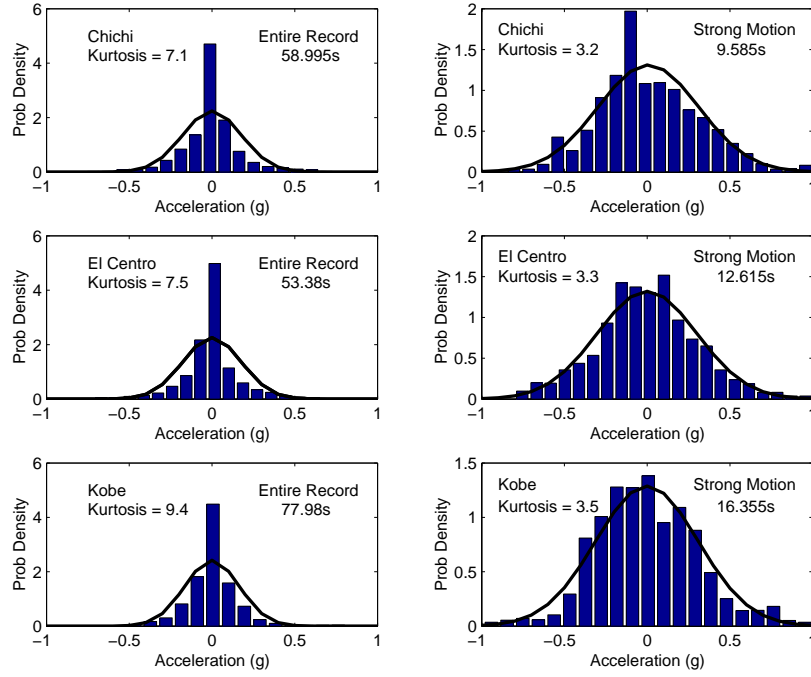


FIG. 1: Comparison of the Gaussianity for the strong motion duration of an earthquake record with the entire record.

3. Illustrative Examples

In order to demonstrate application of the methodology several illustrative examples are presented. These include the investigation of the effect of non-Gaussianity on the Slepian prediction, the ability of the approach to predict the response of linear and nonlinear oscillators, a predictive damage model for a reinforced concrete (RC) three-member frame, and the drift-based reliability of the same three member RC frame. The structural reliability indices of the frame were computed based on transient drift predicted by the Slepian model as well as from the actual data.

3.1 Strong Ground Motion

In order to determine if only the strong motion portion or the entire ground acceleration record should be used, the Slepian model prediction for each was calculated. Figure 2 presents Slepian predictions based upon the total record and the strong motion portion. For each prediction, the numerical covariance and its derivatives from the time series were used. Inspection of Figure 2 leads to the conclusion that the Slepian prediction using only the strong motion portion is only slightly better than the prediction using the entire record. There may be several reasons for this: (1) For the lower level crossings, both the strong motion portion and the entire record have adequate data to obtain the mean values, so there may not be much difference between these predictions; (2) The higher level crossings contain much of the same data for calculation of the mean values, since the strong motion portion contains almost all the higher level crossing data. Figure 3 and Figure 4 present the covariance and its derivative for the strong motion portion and the total record. Note that the covariance function is normalized by the variance of the time series, and its derivatives are normalized by the standard deviation of the derivatives of the time series. Inspection of the figures leads to the conclusion that the normalized covariance of the strong motion portion is almost the same as that of the total record and there does exist some difference between the normalized derivatives of the covariance for the strong motion portion versus the total record. However, it is not significant. Recall that the accuracy of the Slepian prediction relies heavily on the covariance function. The fact that the covariance function for the strong motion portion and the entire record are almost the same provides the explanation for Figure 2.

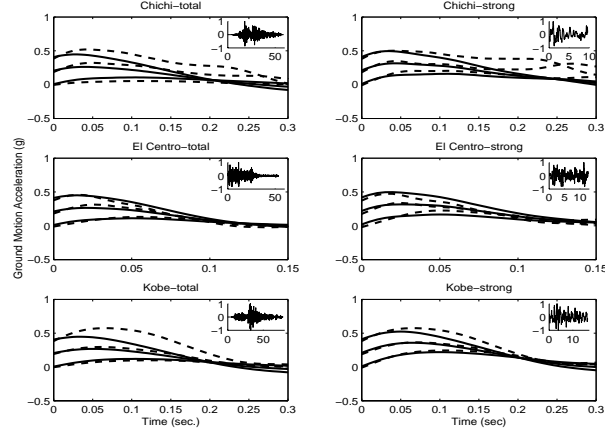


FIG. 2 Slepian Prediction for Total Record versus the Strong Motion Portion

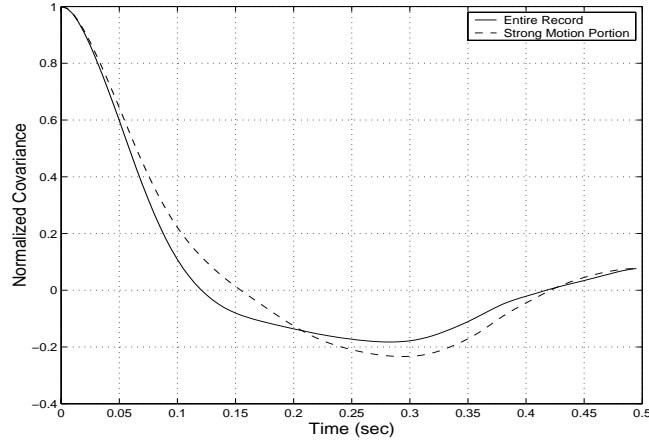


FIG. 3 Normalized Covariance Function

3.2 Linear and Non-Linear Oscillator Response

The response prediction of several different linear oscillators was investigated assuming 5% critical damping in each case and the only the natural period of the models were varied. The Slepian predictions for two different linear oscillators excited by the six different earthquakes identified in Table 1 are shown in Figure 5. It is evident that the accuracy of the predictions varies depending on the particular earthquake used to excite the oscillator, and that the accuracy degenerates for the higher level crossings. This is the result of the number of points in each event decreasing as the specified level crossing is increased. The Slepian model predictions of the mean response behavior appear to fit reasonably well with the mean response obtained directly using the data. For all combinations of oscillator and earthquake the prediction of the expected value above zero-crossing was good.

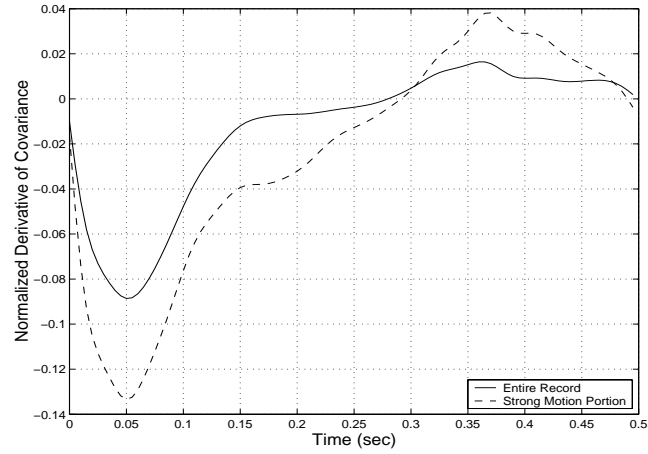


FIG. 4 Normalized Derivative of the Covariance Function

TABLE 1: Details of the earthquakes used in this study

Ground Accelogram and Oscillator Examples				
Earthquake Name	Year	Magnitude	Total Duration of Accelogram (sec.)	Strong Motion Duration from equation (4) (sec.)
Chi Chi, Taiwan	1999	7.3	59.0	9.57
El Centro, USA	1940	7.1	53.4	12.62
Michioacan, Mexico	1985	8.1	156.3	13.20
Duzce, Turkey	1999	7.4	86.2	11.22
Llolleo, Chile	1985	8.0	99.9	44.80
Kobe, Japan	1995	7.2	78.0	16.36
Three-story RC Frame Example				
North Palm Springs	1986	6.0	59.98	4.92

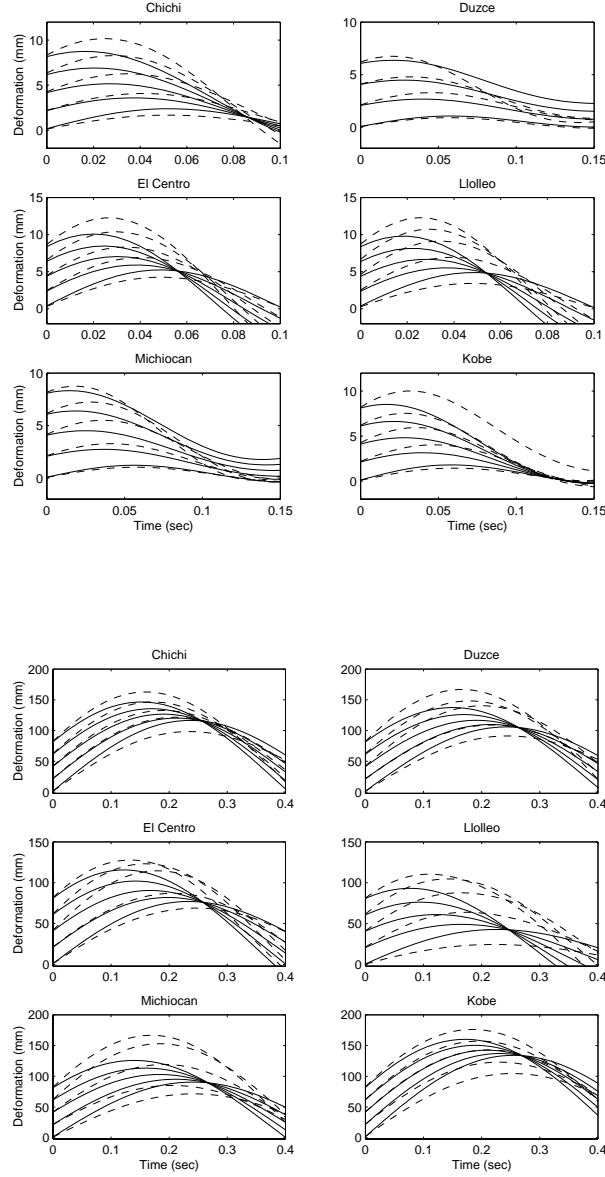


FIG 5: Slepian model prediction of the expected value of the response above level crossings for linear oscillators having periods of 0.2 and 1.0 seconds (from top to bottom) subjected to the six earthquake records presented in Table 1. Dashed line = Data, Solid line = Slepian model prediction.

Recall that, in order to improve the prediction, the Slepian model relies on the derivatives of the time series at the level up-crossing. The expected value form of the Slepian method is based upon the assumption that the derivatives of the time series follow a Rayleigh distribution, which has been demonstrated to be correct for a Gaussian process. For a non-Gaussian time series the derivatives at a level crossing have been shown to follow a Weibull distribution. To confirm this for a single record, Figure 6 presents the survivor

functions of the derivatives at zero up-crossing for the El Centro ground acceleration record and the survivor functions for the theoretical Weibull and Rayleigh distribution. It can be seen that the derivatives at zero-crossing follow a Weibull distribution with shape parameter $\kappa=1.13$, which is almost an exponential distribution. Recall that the Rayleigh distribution is a special case of the Weibull distribution with shape parameter $\kappa=2.0$. In order to show this a little more clearly, Figure 7 presents the shape parameter for forty (40) earthquake chosen records from around the world, whose details are presented in Table 2. One can see that they vary significantly and all have κ values significantly less than 2.0.

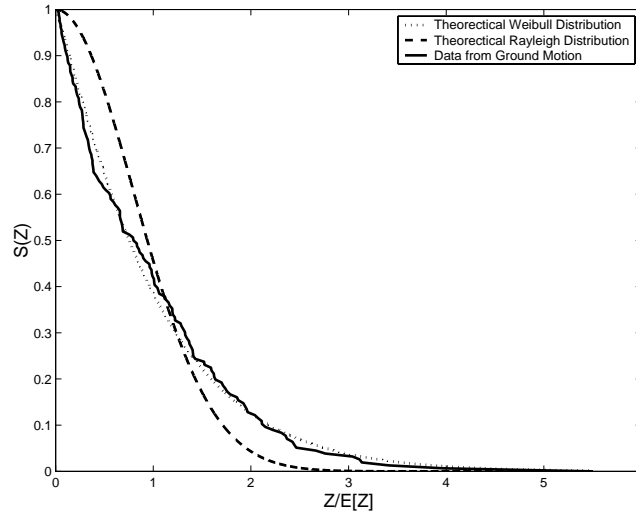


FIG. 6 Survivor Functions for Slopes at Zero Up-crossing for the El Centro Ground Motion

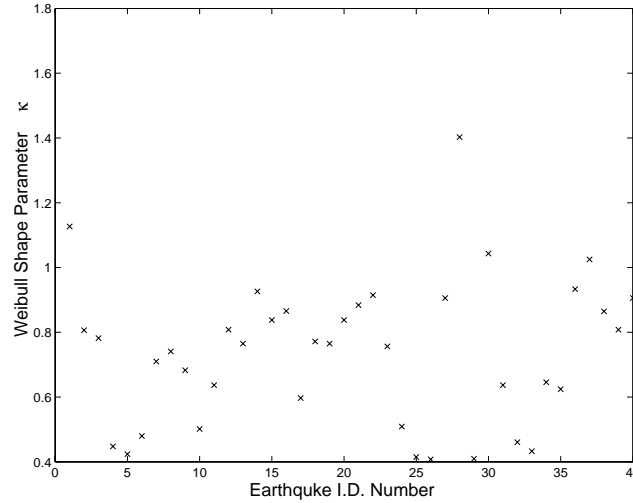


FIG. 7 Weibull Shape Parameter κ for Slopes at Zero Up-crossing for Forty Earthquakes

TABLE 2: Earthquake Details for Forty Records Used in this Study

	File	Earthquake Name	DT	Total	Strong	Damage	Kurtosis	Kurtosis
	Name			Duration	Duration	Index	strong	Total
1	LA01	Imperial Valley, El Centro, 1940	0.020	53.380	14.000	0.592	3.376	7.485
2	LA07	Landers, Barstow, 1992	0.020	79.900	2.980	0.532	3.149	17.411
3	LA11	Loma Prieta, Gilroy, 1989	0.020	39.940	1.620	0.286	2.427	17.636
4	LA13	Northridge, Newhall, 1994	0.020	59.980	4.940	1.080	3.020	28.494
5	LA19	North Palm Springs, 1986	0.020	59.980	4.920	0.267	3.940	30.789
6	LA21	Kobe, 1995	0.020	59.980	1.560	0.483	2.141	30.097
7	LA29	Tabas, 1974	0.020	49.900	8.540	0.326	3.093	11.394
8	LA31	Elysian Park (sim)	0.010	29.990	9.260	0.636	3.688	11.794
9	LA37	Palos Verdes (sim)	0.020	59.980	1.520	1.044	2.059	24.740
10	SE01	Long Beach, Vernon CMD Bldg	0.010	39.050	1.860	0.821	1.958	13.162
11	SE03	Morgan Hill, Gilroy, 1984	0.020	59.980	12.540	0.663	3.708	12.153
12	SE05	West, Wahington, Olympia, 1949	0.020	79.900	13.040	0.558	3.288	11.651
13	SE11	Puget Sound, Wa., Olympia, 1949	0.020	81.820	4.840	0.467	2.833	19.577
14	SE15	Eastern Wa., Tacoma County, 1949	0.020	59.980	13.100	0.462	3.623	7.712
15	SE17	Liolleo, Chile 1985	0.025	99.875	44.800	1.231	3.660	7.591
16	SE19	Vinadel Mar, Chile, 1985	0.025	99.875	33.100	1.214	2.929	5.904
17	SE21	Mendocino, 1992	0.020	59.980	3.340	0.596	3.144	40.629
18	SE23	Erzincan 1992	0.005	20.755	3.865	0.950	3.031	11.764
19	SE27	Seattle 1965	0.020	81.820	4.840	0.467	2.833	19.576
20	SE29	Valpariso 1985	0.025	99.875	44.800	1.231	3.661	7.591
21	SE33	Deep Interplate (sim)	0.020	79.900	10.080	0.255	3.318	10.329
22	SE35	Miyagi-oki 1978	0.020	79.900	10.500	0.646	2.905	16.431
23	SE37	Shallow Interplate (sim)	0.020	79.900	15.460	0.516	3.399	12.797
24	BO01	Hanging Wall, (sim)	0.010	29.990	4.920	0.401	4.837	27.532
25	BO03	Foot Wall (sim)	0.010	29.990	1.420	0.308	2.523	30.045
26	BO05	New Hampshire 1982	0.005	19.225	0.270	0.060	2.446	40.454
27	BO07	Nahanni 1985	0.005	20.335	1.505	0.197	3.600	12.598
28	BO13	Saguenay, 1988	0.005	17.725	14.905	0.310	4.751	5.206
29	LA41	Coyote Lake 1979	0.010	26.810	1.090	0.319	1.591	24.721
30	LA45	Kern 1952	0.020	78.580	15.460	0.822	2.973	7.482
31	LA49	Morgan 1984	0.020	59.980	12.540	0.663	3.708	12.153
32	LA51	Parkfield 1966	0.020	43.900	7.860	0.321	6.667	33.628
33	LA57	San Fernando 1971	0.020	79.420	7.520	0.296	3.363	26.238
34	LA59	Whittier 1987	0.020	39.940	2.160	0.672	2.179	25.235
35	Chichi	Chichi	0.005	58.995	9.565	1.403	3.218	7.111
36	Duzce	Duzce	0.005	86.170	11.220	1.238	3.306	8.874
37	Michiocan	Michiocan	0.030	156.300	13.200	1.533	3.018	10.914
38	LA23	Loma Prieta 1989	0.010	24.950	6.560	0.653	2.257	7.419
39	SE25	Olympia 1949	0.020	79.900	13.040	0.558	3.288	11.651
40	BO27	Nahanni Station1, 1985	0.005	20.335	1.505	0.197	3.600	12.598

Because slopes at level up-crossings play an important role in the Slepian methodology, it is logical to investigate the relationship between the Weibull shape parameter for slopes at zero-crossing and the resulting Park-Ang damage index. Figure 8 presents the Weibull shape parameter κ for slopes at the zero up-crossing for the ground acceleration record versus the Park-Ang structural damage index, D , calculated using an arbitrary elasto-plastic oscillator.

The linear regression is also presented in the same figure. Inspection of Figure 8 tells us that the Weibull parameter κ increases slightly for larger damage indices although there is clearly significant scatter. The dashed line in Figure 8 shows the value of κ that corresponds to a Park-Ang damage index of 1.0, at least in a least squared sense. Figure 9 presents that point ($\kappa=0.79$) as the left most point at an abscissa value of zero, since it is for zero-crossing. Repeating the procedure for level crossings of 0.1, 0.2, 0.3, and 0.4 one can see that as the level crossing increases the shape parameter for the slopes of the ground acceleration record at that level crossing tends to increase. Note that the dashed lines in Figure 9 represent +1 and -1 standard deviation for the predicted value based on the linear regression (Neter, et. al, 1996). The trend is clear since they increase also as the level crossing increases. This observation may allow prediction of expected damage levels because each of the points in Figure 9 corresponds to $D=1.0$.

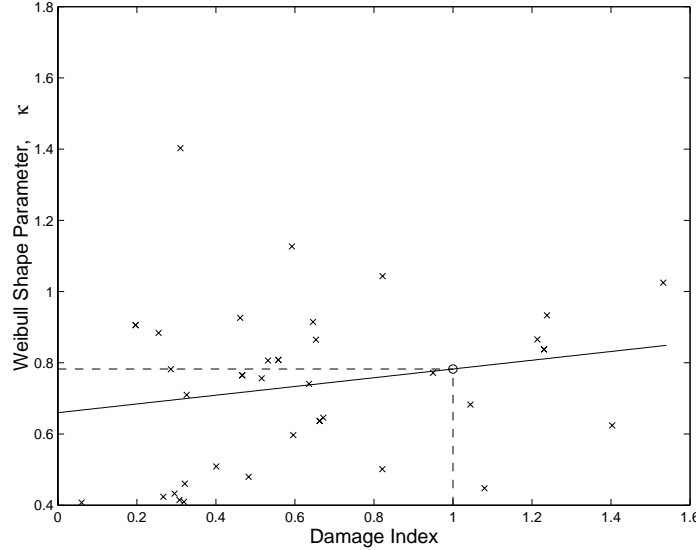


FIG. 8 Weibull Shape Parameter κ for Slopes at Zero Up-crossing Versus Damage Index for Forty Earthquakes

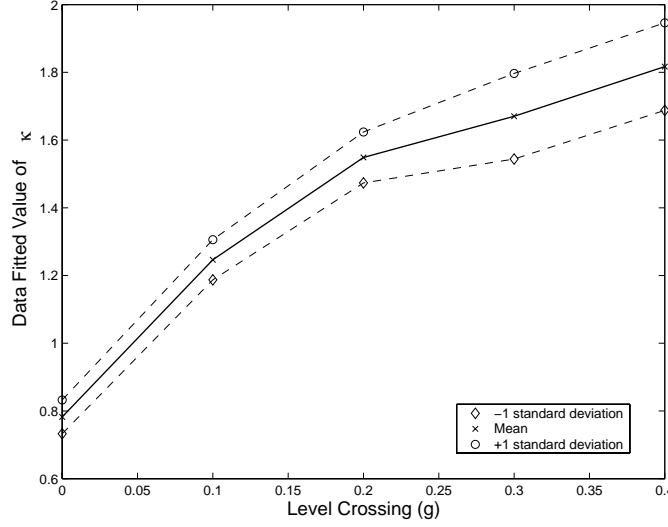


FIG. 9 Weibull Shape Parameter κ at Different Level Crossings

3.3 Three-Member Reinforced Concrete Frame

A three-member RC frame was detailed using typical code procedures. A software package available through the multi-disciplinary center for Earthquake Engineering, IDARC (Inelastic Damage Analysis of Reinforced Concrete) was used to estimate the response of the structure to ground motion excitation. Figure 10 presents a comparison between the expected value prediction at zero-crossing and at a higher level crossing ($\sim 61\text{mm}$) for the 1986 North Palm Springs earthquake. The prediction is very good at the zero crossing immediately following, i.e. 0.4 sec, but there is some discrepancy later. The difference between the simulated data and the Slepian prediction for the higher level crossing is approximately 13.5%. This is expected since there are only a handful of data points available at this higher level. (see “drift from IDARC model” in upper right corner of Figure 10). Now, consider the structural reliability problem in engineering which can be solved using a first order approximation known as the FORM.

The calculation of the reliability index can be accomplished by considering the following limit state function

$$G = C - D \quad (31)$$

where, G is the limit state surface, C is the structural capacity, and D is the seismic demand on the structure. The structure is considered safe for all non-negative values of G , and fails for negative values of G . The reliability index, β , is estimated from the first order reliability method (FORM) using the following expression

$$\beta = \Phi^{-1}(1 - p_f) \quad (32)$$

where, Φ^{-1} is the inverse of the standard normal distribution function and p_f is the failure probability. The capacity, C , in Equation (31) was given in terms of inter-story drift by Dymiotis et al. (1999) as lognormal having a mean of 6.6% and a coefficient of variation (COV) of 31%. This was based upon their analysis of 76 tests from around the world conducted between 1974 and 1998. The seismic demand, D , in equation (31) can be determined from the Slepian model prediction or from data and its mean is established as from the expression

$$\mu_D = \max \{E[x_{uz}(t)]\} \quad (33)$$

where, μ_D is the mean value of the seismic demand. The COV of D is not explicitly known for earthquakes in general since it is typically dominated by the ground motion itself. However, the statistical distribution is well described by a Weibull distribution (Niedzwecki et al. 2000; van de Lindt and Goh 2003) having a mean calculated from the expression in equation (33). Figure 11 presents the variation in the reliability index as the COV of the seismic demand varies. The mean of the seismic demand from both the Slepian model as well as the time series data is shown. One can argue that the COV of the seismic demand is quite large, in general in excess of 100% (see for example Ellingwood et al. 1980). Moreover, it is unlikely that a COV would exceed 200%. By bounding utilizing these two COV values, this provides a bounded solution for the reliability index. Taking the lower bound as conservative the Slepian prediction estimates $\beta = 1.51$ compared to the actual value of $\beta = 1.43$, a difference of 5.6%. It was concluded that for moderately nonlinear RC structures, particularly frames, that the reliability indices computed using a Slepian predictive model are non-conservative by about 5 to 10%. This was further investigated and verified by results not presented herein, and it was found that the results were similar regardless of the ground motion selected.

4. Conclusions

Based on the analyses conducted in this study it can be concluded that the Slepian Type I model does not provide a significantly better prediction when using only the strong ground motion portion of a record, even though that portion of the record is very nearly Gaussian. The prediction was significantly better for the response of a linear oscillator compared to that of a nonlinear oscillator, elasto-plastic or bilinear. This was caused by a shift in equilibrium position for the highly nonlinear oscillators and could be corrected by applying an approximate correction.

Based on the analysis of the several simple nonlinear oscillators and their response to a suite of forty (40) earthquakes a trend between the level crossing and Weibull shape parameter for the response was observed. This observation, once generalized, could provide a statistical solution to the inverse problem, i.e. quantifying the distribution of the potential response for a structure near failure. Understanding that statistical distribution of the accelogram peaks play a role, may help to determine which ground motions cause damage to engineered structures. At this stage, it can be concluded that for basic

nonlinear oscillators the lower the Weibull shape parameter for the ground motion peaks the lower the probability of damage. However, this preliminary conclusion is based on analysis with only forty earthquakes, and a limited number of nonlinear oscillators.

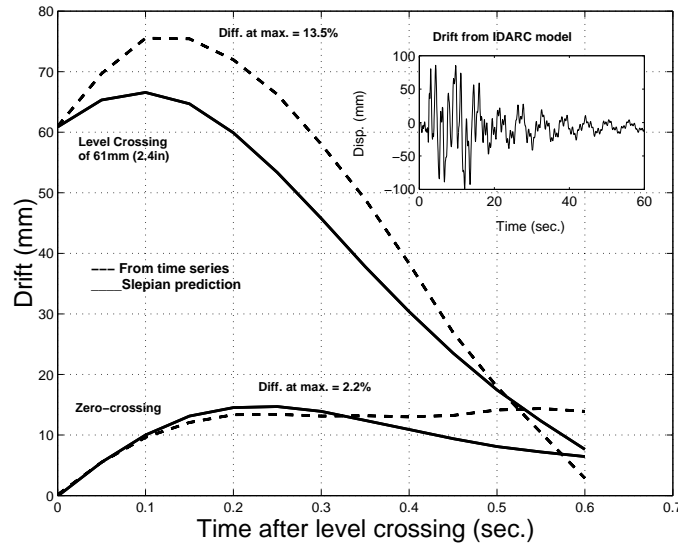


FIG 10: Slepian prediction of drift above zero-crossing and an significantly higher level-crossing for the three-story reinforced concrete frame versus data generated using IDARC version 3.0.

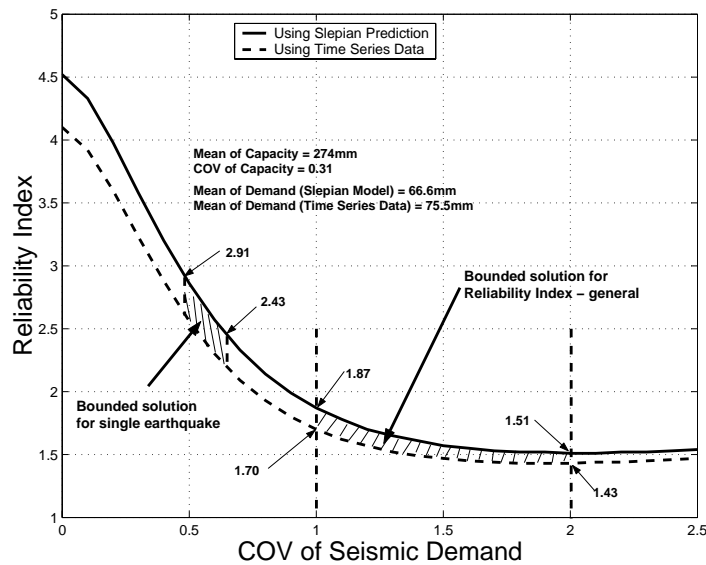


FIG 11: Reliability index of the three-story frame using the results of the Slepian prediction as the mean of the frames capacity. The reliability indices were calculated using the limit state function $G = C - D$ and solved via the first order reliability method (FORM) approximation.

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